

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/07
Paper 7 Probability \& Statistics 2 (S2)
October/November 2007
1 hour 15 minutes
Additional Materials: Answer Booklet/Paper Graph Paper List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Isaac claims that $30 \%$ of cars in his town are red. His friend Hardip thinks that the proportion is less than $30 \%$. The boys decided to test Isaac's claim at the $5 \%$ significance level and found that 2 cars out of a random sample of 18 were red. Carry out the hypothesis test and state your conclusion. [5]

2 In summer the growth rate of grass in a lawn has a normal distribution with mean 3.2 cm per week and standard deviation 1.4 cm per week. A new type of grass is introduced which the manufacturer claims has a slower growth rate. A hypothesis test of this claim at the $5 \%$ significance level was carried out using a random sample of 10 lawns that had the new grass. It may be assumed that the growth rate of the new grass has a normal distribution with standard deviation 1.4 cm per week.
(i) Find the rejection region for the test.
(ii) The probability of making a Type II error when the actual value of the mean growth rate of the new grass is $m \mathrm{~cm}$ per week is less than 0.5 . Use your answer to part (i) to write down an inequality for $m$.

3 (i) Explain what is meant by the term 'random sample'.
In a random sample of 350 food shops it was found that 130 of them had Special Offers.
(ii) Calculate an approximate $95 \%$ confidence interval for the proportion of all food shops with Special Offers.
(iii) Estimate the size of a random sample required for an approximate $95 \%$ confidence interval for this proportion to have a width of 0.04 .

4 The cost of electricity for a month in a certain town under scheme $A$ consists of a fixed charge of 600 cents together with a charge of 5.52 cents per unit of electricity used. Stella uses scheme $A$. The number of units she uses in a month is normally distributed with mean 500 and variance 50.41.
(i) Find the mean and variance of the total cost of Stella's electricity in a randomly chosen month.

Under scheme $B$ there is no fixed charge and the cost in cents for a month is normally distributed with mean 6600 and variance 421 . Derek uses scheme $B$.
(ii) Find the probability that, in a randomly chosen month, Derek spends more than twice as much as Stella spends.

5 The length, $X \mathrm{~cm}$, of a piece of wooden planking is a random variable with probability density function given by

$$
\mathrm{f}(x)= \begin{cases}\frac{1}{b} & 0 \leqslant x \leqslant b \\ 0 & \text { otherwise }\end{cases}
$$

where $b$ is a positive constant.
(i) Find the mean and variance of $X$ in terms of $b$.

The lengths of a random sample of 100 pieces were measured and it was found that $\Sigma x=950$.
(ii) Show that the value of $b$ estimated from this information is 19 .

Using this value of $b$,
(iii) find the probability that the length of a randomly chosen piece is greater than 11 cm ,
(iv) find the probability that the mean length of a random sample of 336 pieces is less than 9 cm . [4]

6 The random variable $X$ denotes the number of worms on a one metre length of a country path after heavy rain. It is given that $X$ has a Poisson distribution.
(i) For one particular path, the probability that $X=2$ is three times the probability that $X=4$. Find the probability that there are more than 3 worms on a 3.5 metre length of this path.
(ii) For another path the mean of $X$ is 1.3 .
(a) On this path the probability that there is at least 1 worm on a length of $k$ metres is 0.96 . Find $k$.
(b) Find the probability that there are more than 1250 worms on a one kilometre length of this path.

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